QM1

a) \[ \langle \alpha | \beta \rangle = e^{-\frac{\alpha^2}{2} - \frac{\beta^2}{2} + \alpha^* \beta} \]

So the pure coherent states are never orthogonal for any choices of \( \alpha \) and \( \beta \).

b) 

1

\[ \langle \psi_\pm | \psi_\pm \rangle = \frac{N_\pm^2(\beta)}{2} (1 + 1 \pm \langle \beta | \overline{\beta} \rangle \pm \langle \overline{\beta} | \beta \rangle) \]

Note that \( \langle \beta | \overline{\beta} \rangle = \langle \overline{\beta} | \beta \rangle \). Therefore \( N_\pm = \frac{1}{\sqrt{1 \pm e^{-2|\beta|^2}}} \), clearly \( N_\pm(\infty) = 1 \).

2

We verify that \( \langle \psi_\pm | \psi_\pm \rangle = 1 - 1 + \langle \beta | \overline{\beta} \rangle - \langle \overline{\beta} | \beta \rangle = 0 \).

3

\[ \langle n | \psi_\pm \rangle = \frac{1}{\sqrt{2n!(e^{\beta^*} - e^{-\beta})}} \beta^n \pm (-\beta)^n \]

\[ P_\pm(n) = \frac{e^{-|\beta|^2}}{2n!(1 \pm e^{-2|\beta|^2})} \beta^n \pm (-\beta)^n \]

\[ \langle \beta^n (\pm 1) \rangle = \frac{e^{-|\beta|^2}}{n!(1 \pm e^{-2|\beta|^2})} \beta^{2n}(1 \pm (-1)^n) \]

1Note that the summation given in the hint should start from \( n = 0 \), i.e. \( \Sigma_{n=0}^{\infty} \frac{\beta^n}{n!} = e^\beta \).
(c)

1

\[ [K_1, K_2] = \frac{1}{4i} ([a^2, a^{\dagger 2}] - [a^{\dagger 2}, a^2]) \]
\[ = \frac{1}{2i} [a^2, a^{\dagger 2}] \]
\[ = -4iK_3 \]

2

By uncertainty principle we know \( \Delta K_1 \Delta K_2 \geq \frac{1}{2} |[K_1, K_2]| = |2\langle K_3 \rangle| \).

3

We calculate

\[ \Delta^2 K_1 = \langle K^2 \rangle - \langle K \rangle^2 \]
\[ = \frac{1}{2} (2|\gamma|^2 + 1) \quad (1) \]

Following the same calculation we obtain the identical result for \( \Delta^2 K_2 \). Therefore \( \Delta K_1 \Delta K_2 = |2\langle K_3 \rangle| = |\gamma|^2 + \frac{1}{2} \)
QM2

a)  
Note that the correct form of the wave equation should actually be $\psi(\vec{x}) = e^{\frac{i}{\hbar}(p_x x + p_y z)} \phi(y) \chi_{\pm}$. Plugging in the time-independent Schrödinger equation, we get $\frac{1}{2m}(\hat{p}_x^2 + \hat{p}_y^2 + (p_x + 2By)^2)\phi \mp \frac{\hbar B}{2mc} \phi = E\phi$.

b)  
Rearranging the Schrödinger equation, we immediately recognize the form of the equation as that of a 1D SHO with angular frequency $\omega = \frac{qB}{mc}$ and a horizontal shift of $-\frac{qC}{qB} = y_0$ to the right:

$$\left(\frac{\hat{p}_y^2}{2m} + \frac{1}{2} m \omega^2 (y - y_0)^2\right) \phi = \left(E - \frac{p_z^2}{2m} \pm \frac{\hbar \omega}{2}\right) \phi$$

We can thus read off the spectra $E = \hbar \omega(n + \frac{1}{2}) + \frac{\hat{p}_y^2}{2m} \mp \frac{\hbar \omega}{2} = \hbar \omega(n + \frac{1}{2} \pm \frac{1}{2}) + \frac{\hat{p}_y^2}{2m}$.

c)  
In the new gauge, the momentum terms are adjusted to $p_x + \frac{qBy}{qB} = \partial_x \Lambda$, $\hat{p}_y - \frac{qC}{qB} \partial_y \Lambda$, and $p_z - \frac{qC}{qB} \partial_z \Lambda$. However, upon evaluating these terms on the transformed wave function, $\psi'$, the additional components involving $\Lambda$ all cancel out, resulting in the original Schrödinger equation.

d)  
Based on the given requirement, it is evident that a suitable choice for the vector potential is $Bx\hat{j}$, which can be obtained through the gauge transformation $(By, Bx, 0)$ from the original vector potential, i.e. $\Lambda(\vec{x}) = Bxy$. In this scenario, the momentum term in the Schrödinger equation simplifies to $\frac{1}{2m}(\hat{p}_x^2 + \hat{p}_y^2 + \frac{qB^2}{qC} (x - x_0)^2)$, where $x_0 = \frac{p_y c}{qB}$. And the corresponding wavefunctions are given by

$$\psi'(\vec{x}) = e^{\frac{i}{\hbar}(p_y y + p_z z)} \phi'(x) \chi_{\pm}$$

Consequently, we observe an oscillator-like behavior that is confined to the $x$ direction but may exhibit shifts in the $y$ direction.